Rates, Ratios, and Proportions

Rates:
Comparisons of two unlike quantities. Words such as in, per, by, for, each, etc. are used to express rates.

i.e. 5 apples for $2.30
3 min. per person

Rates are often given in quantities other than 1, but the unit rate or cost for 1 unit can be calculated.

Unit Rates:
If you get 5 apples for $2.30, then $2.30 ÷ 5 = $0.46 per apple. Use this method to do comparison shopping between stores.

Ratios:
Comparisons of two like quantities when expressed in identical units.

i.e. half of a pie. 1:2, 50%, or $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$

Ratios are expressed in three ways: **Always Simplify to lowest terms (reduce).

1) The colon a:b OR
2) Using "To" OR
3) The fraction form (division bar) a/b or $\frac{a}{b}$

Equivalent Ratios can be:
-Divided by the same numbers to create equal quotients
-Reduced to equal ratios (fractions)

Ratios have a **multiplicative property**. This means that the numerator and denominator, when **multiplied** or **divided** by the same number, create equal proportions (fractions).

i.e. $\frac{12^{\div 6}}{18^{\div 6}} = \frac{2}{3}$, $\frac{2^{\times 7}}{3^{\times 7}} = \frac{14}{21}$

This property does NOT work for **addition** and **subtraction**

i.e. $\frac{6^{-3}}{9^{-3}}$ is not equal to $\frac{3}{6}$ because 6/9 is two thirds and 3/6 is one half.

Ratios Vs. Fractions:

Ratios can be Part-to-Part or Part-to-Whole

Types of Ratio	Example	Can be written as a fraction accurately?
Part-to-Part	Girls to Boys in a class 4:6 or 2:3	No, because it compares the part of the class that is girls to the part of the class that is boys.
Part-to-Whole	Using example above with 4 girls and 6 boys in a class Ratio of Girls to Class 4:10 or 2:5	Yes, because the ratio is now a part to whole. Girls are part of the class and there are 10 students total (4+6=10).
Ratios with Different Units	3 desks for 4 students Miles of driving per gallon of gas Dollars made (wage) per hour of work	No, because fractions compare part to whole of the same units.

Proportion:
An equality between two equivalent ratios or rates.

Proportional Series:
The series of numbers forming the rows or columns in the table of values of a proportional situation.

Proportional Quantities:
Quantities that form a proportion.

Proportional Situations:
A situation resulting in equivalent ratios or rates. In a proportional situation, rates and ratios must be equivalent.

Table of Values:

In a table of values of a proportional situation, the numbers in the (x) and (y) columns form equal proportions. They create a proportional series as below.

Independent Variable (x)	Time (h)	0	1	2	3	4	5	6	10
Dependent Variable (y)	Wage ($)	0	3	6	9	12	15	18	30

**In the above table of values the numbers in the (y) column are found by multiplying the numbers from the (x) column by a single number called the constant of proportionality. In this example, the constant of proportionality is $3 per hour.

To find the constant of proportionality from a table of values you can:

a) Find the **unit rate** $30 for 10 hours = $3/hour OR

b) Use the formula K= y/x where **K** is the Constant of proportionality so K= 6/2, K=3

c) Take any two pairs of coordinates from above table i.e. (6,18) and (10,30) and find the **rate of change** (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{30 - 18}{10 - 6}$$

$$= \frac{12}{4}$$

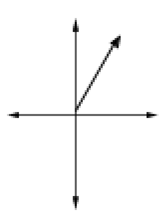
$$= \$3\ /\text{hour}$$

Proportionality and Graphs:

A graph of a proportional situation has two important features:

1: It is a straight line and

2: It passes through the origin (point 0,0)



Equivalence:

If the ratio a : b is equal to the ratio c : d then the proportion can be written as:

$$a : b = c : d \quad \text{OR} \quad \frac{a}{b} = \frac{c}{d}$$

The ratio is read as: a is to b as c is to d

In these cases the numbers a, b, c, and d are said to be proportional.
For example 1 : 2 = 4 : 8 OR 1 is to 2 what 4 is to 8. They both represent half.

Finding a missing term in a proportion:

Finding a missing term in a proportion can be done in one of four ways:

a) **Use the graph**: Look at the graph to locate the missing x or y value. Remember that each ordered pair from the table of values is a point on the graph
b) **Find the unit rate** or the cost of 1 item and then multiply by the appropriate number of items
c) **Factor of change**. If you buy twice as much, your bill should be twice as much. (often used with the table of values)
d) **Cross Product**: Create a proportion (two equivalent ratios or fractions and cross multiply to find the missing term. This method is often the most efficient and can be used for many mathematical situations.

i.e. If you buy 36 stamps it costs you \$18.72. How much would 54 stamps cost?

$$\frac{36}{\$18.72} = \frac{54}{\$X}$$
$$36X = 54(18.72)$$
$$\frac{36X}{36} = \frac{1010.88}{36}$$
$$X = \$28.08$$

OR

$$\frac{36}{54} = \frac{\$18.72}{\$X}$$
$$54(18.72) = 36X$$
$$\frac{1010.88}{36} = \frac{36X}{36}$$
$$\$28.08 = X$$

**Notice that no matter which of these two ways that you set up the proportion, you multiply the same numbers, 36 times (X) and 18.72 times (54).