Number Sense and Decimal Unit Notes

Table of Contents:

Place Value and Rounding:

Place Value Chart

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thoussandths	Ten Thousandths
4	6	3	8	2	3	•	6	7	2	6

Rounding

If the digit to the right of the one you are rounding is 5 or more, add 1 to the digit being rounded.

If the digit to the right of the one you are rounding is less than 5, do not change the digit being rounded.

i.e. Round 3273 to the nearest hundred = 3300

ie. Round 6.931 to the nearest hundredth = 6.93

Face Value, Place Value, Total Value

These terms allow us to refer to the value of a digit placed in the context of a number.

i.e. Look at the digit 7 in the number 6**7**8.

Face Value: The digit we see : 7
Place Value: The place value of the underlined digit: tens
Total Value: The value of the number in the context. Here we have 7 tens so the total value is 7 x 10 or 70.

i.e. Look at the digit 4 in the number 856.3**4**

Face Value: The digit we see : 4
Place Value: The place value of the underlined digit: hundredths
Total Value: The value of the number in the context. Here we have 4 hundredths so the total value is 4 x .01 or 0.04 (four hundredths).

Standard and Expanded Form:

Standard Form: How we normally see numbers.
i.e. 456, 879.34,

Expanded Form: Numbers written as a product of face value times place value.

i.e $456 = 4 \times 100 + 5 \times 10 + 6 \times 1$

i.e. $879.34 = 8 \times 100 + 7 \times 10 + 9 \times 1 + 3 \times 0.1 + 4 \times 0.01$

Factoring:

Factor: A number that divides equally into another number. Factors are less than or equal to the number.

Prime Number: A number with exactly two different factors, one and itself.

Composite Number: A number with more than two different factors.

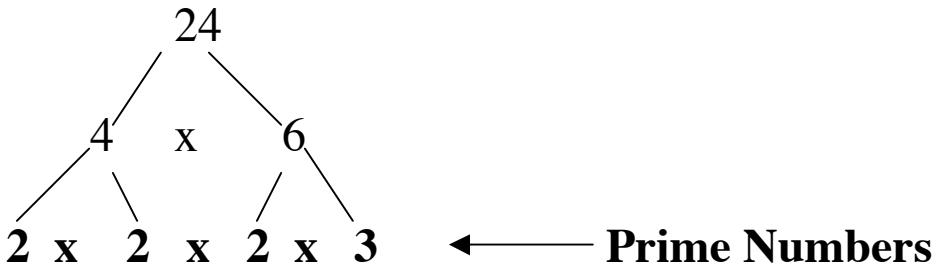
Prime numbers up to 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Divisibility Rules: Divisibility Rules Help us find the factors
A number is divisible by:

2 if it ends in 0, 2, 4, 6, or 8.
3 if the sum of the digits is divisible by 3
4 if the last two digits are divisible by 4
5 if it ends in 0, 5
6 if it is divisible by 2 and 3
8 if the last three digits are divisible by 8
9 if the sum of the digits is divisible by 9.
10 if it ends in 0.
12 if it is divisible by 3 and 4.

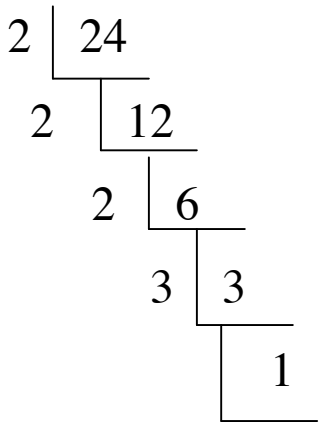
Prime Factorization: (Factor Tree and Staircase)

Factor tree: start with any pair of factors and keep breaking down until all numbers at the bottom of the branches are prime.



24 = 2 x 2 x 2 x 3

Staircase: At each step divide by a prime number. Start with the smallest and keep going until you get down to 1.



$24 \div 2 = 12 \div 2 = 6 \div 2 = 3 \div 3 = 1$

24 = 2 x 2 x 2 x 3

Greatest Common Factor and Lowest Common Multiple:

Factor: A whole number that divides exactly into another number. Factors are less than or equal to the number

Multiple: The product of a whole number and another whole number. Multiples are at equal or greater than the number. Multiples of 4 would be: 4, 8, 12, 16, 20, 24, etc. When you count by 4's you are saying all the multiples of 4.

GFC (Greatest Common Factor): The highest number that divides exactly into two or more numbers.

LCM (Least Common Multiple or Lowest Common Multiple): The smallest number that is a multiple of two or more numbers.

This is how you likely found GCF and LCM in elementary school:

GFC (Greatest Common Factor):

Example 1: Determine the GCF of 12 and 30

Step 1: Find factors of 12 and 30

12 = 1 x 12
12 = 2 x 6
12 = 3 x 4

30 = 1 x 30
30 = 2 x 15
30 = 3 x 10
30 = 5 x 6

Step 2: Highlight the factors they have in common.
Step 3: The GREATEST CF is 6

LCM (Least Common Multiple or Lowest Common Multiple):

Example 1: Find the LCM of 3 and 5

Step 1: Find the multiples of 3 and 5

3	5
6	10
9	15
12	20
15	25
18	30
21	
24	
27	
30	

The Least Common Multiple of 3 and 5 is 15, because 15 is a multiple of 3 and also a multiple of 5. Another common multiple of 3 and 5 is 30 (but it is NOT the LCM).

When you are asked to find the GCF or LCM it is referring to at least two different numbers but it could be three or more. You are asked to find "common" factors or multiples.

A new Strategy for finding the GCF and LCM using a venn diagram is:

Step 1: Use the factor staircase to determine all the prime factors of each of the numbers.

Step 2: Place the numbers in the venn diagram. If the factor is common, place it in the overlapping section. If it is a factor only of one number, place it in the section only for that number.

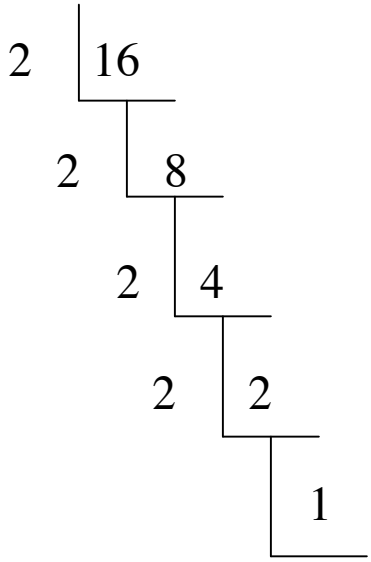
Step 3:

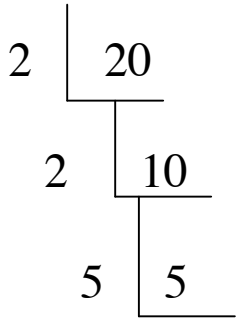
For GCF: multiply together the factors that are in the shared part of the venn diagram. The product is the GCF.

For LCM: Multiply together all the factors that are in the venn diagram. The product is the LCM.

i.e. Find the GCF and the LCM of 16 and 20.

Step 1: Factor staircases for both numbers

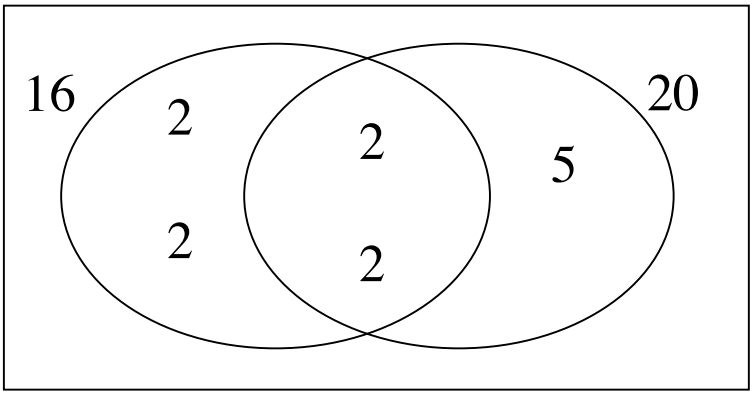




16 = 2 x 2 x 2 x 2 20 = 2 x 2 x 5

Yellow is common, underlined are factors of 16 only, double underlined are factors of 20 only

Step 2: Place number in the venn diagram



GCF: 2 x 2 = 4 (multiply numbers from the center)

LCM: 2 x 2 x 2 x 2 x 5 = 80 (multiply all numbers from the venn diagram)

Greatest Common Factor and Lowest Common Multiple Word Problems:

Use Greatest common factor when you need to break numbers down in to equal sized groups.

Use Lowest Common Multiple when you need to find occurrences of several situations at the same time.

Exponents: Also called the Index or power of a number.

Exponents or powers allow us to show repeated multiplication in a simple way. For example if a singer sold 100 000 CD's, we could say he sold:
10 x 10 x 10 x 10 x 10 CD's or 10^5.

The power of a number shows how many times a number is multiplied by itself.

i.e. The power of 2^4 is 4. It means 2 x 2 x 2 x 2 = 16
We say two to the power of four or two raised to the fourth power. The number 2 is the base and the 4 is the exponent, power, or index.

Exponents can be written in different forms:

Exponential Form	Expanded Form	Standard Form
2^5	2 x 2 x 2 x 2 x 2	32
3^2	3 x 3	9
4^3	4 x 4 x 4	64

Special Exponents: 0 and 1

Exponent of 1:
Any number with an exponent of 1 is equal to itself (the base number)

i.e. $8^1 = 8$
i.e. $2^1 = 2$
i.e. $-3^1 = -3$

Exponent of 0:
Any number with an exponent of zero is equal to 1

i.e. $8^0 = 1$
i.e. $2^0 = 1$
i.e. $-3^0 = 1$
so i.e. $(\text{any number})^0 = 1$

For an explanation of why, look at the pattern below. As you can see, when the base is 4 each time you decrease the exponent by 1, the value of the power is 1/4 as large or divided by 4. So negative exponents turn whole numbers into fractions.

$4^3 = 64$
$4^2 = 16$
$4^1 = 4$
$4^0 = 1$
$4^{-1} = 1/4$ or 0.25

Each step is divided by 4.

$64 \div 4 = 16$
$16 \div 4 = 4$ etc.

$4^{-2} = 1/16$ or 0.0625
$4^{-3} = 1/64$ or 0.015625

Order of Operations (BEDMAS):

Properties of Numbers:

Commutative Property: For addition and multiplication. The order property. It states that changing the order does not change the sum or product.
i.e. $a + b = b + a$ so $4 + 5 = 5 + 4$
i.e. $a \times b = b \times a$ so $2 \times 3 = 3 \times 2$

Does not work for subtraction and division
i.e. $a - b \neq b - a$ so $4 - 2 \neq 2 - 4$
i.e. $a \div b \pm b \div a$ so $4 \div 2 \pm 2 \div 4$

Associative Property: For addition and multiplication. It states that changing the addends does not change the sum. Or changing the groups of factors does not change the product.
i.e. $(a + b) + c = a + (b + c)$ so $(3 + 4) + 5 = 3 + (4 + 5)$
i.e. $(a \times b) \times c = a \times (b \times c)$ so $(3 \times 4) \times 5 = 3 \times (4 \times 5)$

Distributive Property: For Multiplication. Can help you multiply in your head.

i.e. $a(b + c) = ab + ac$ so $2(6 + 3) = 2 \times 6 + 2 \times 3$
**this property is very important for algebra.

**The order in which we perform operations in an expression. The reason we have this in mathematics is so we will all get the same answers to the same expressions.

i.e. 2 + 6 x 2

The correct answer is 14 (2 + 12) but if you did not follow the order of operations you would get 16 (8 x 2).

WHY does it matter: In mathematics the order of operations we follow can be described by an acronym BEDMAS for the order where each letter stands for an operation that needs to be performed. If the question does not include the operation at one step, skip to the next.

B = Brackets
E = Exponents
D **M** Do division and multiplication in the order they appear from left to right like reading a book
A **S** Do adding and subtracting in the order they appear from left to right like reading a book

i.e. 2 + 6 x 2
m 2 + 12
a 14

At each step underline what you need to do and replace the underlined part with the answer on the next line. Each line will simplify the question a bit more. Circle your answer.

i.e. $15 \div (7 - 4) \times 6$

b	$15 \div 3 \times 6$
d	5×6
m	30

i.e. $(4 + 2)^2 - 5$

b	$6^2 - 5$
e	$36 - 5$
s	31

i.e. $3^2 \div 3 + 2 \times (12-10)^2$

b	$3^2 \div 3 + 2 \times (2)^2$
e	$9 \div 3 + 2 \times 4$
d	$3 + 2 \times 4$
m	$3 + 8$
a	11

Decimal Ordering and Rounding:

Ordering decimals means putting decimals in an order either from smallest to largest or largest to smallest depending on the instruction.

How to order Decimals:

i.e. Determine which of the following numbers is largest; 5612.47 or 5612.451

Step 1: Line up the digits by place value: 5612.47
5612.451

Step 2: Look at the digits from left to right to determine the biggest. The bigger the numbers the bigger the decimal, it does not have to do with the length of the decimal number but with the value of the numbers.

Step 3: If the digits are the same, look for the first place that they are different. Here the 5612.4 are all the same. The first place that there is a difference is in the hundredths place. Here 7 hundredths > 5 hundredths so we can determine that 5612.47 > 5612.451.
**Use the same procedure if you have to order more than 2 numbers.

Adding Decimals:

Adding decimals involves lining up the numbers according to place value.

i.e. 0.58 + 0.373

Step 1: Line up the numbers according to place value:

$$\begin{array}{r} 0.580 \\ +\ 0.373 \end{array}$$

Step 2: Add the numbers in the same place value as you do with whole number addition.

$$\begin{array}{r} 0.580 \\ +\ 0.373 \\ \hline 0.953 \end{array}$$

Step 3: Write in the decimal in the proper place according to place value.

Subtracting Decimals:

Subtracting decimals involves lining up the numbers according to place value.

i.e. 7.89 – 4.712

Step 1: Line up the numbers according to place value:

$$\begin{array}{r} 7.89 \\ -\ 4.712 \end{array}$$

Step 2: Subtract the numbers in the same place value as you do with whole number subtraction. Borrow as needed.

$$\begin{array}{r} 7.890 \\ -\ 4.712 \\ \hline 3.178 \end{array}$$

Step 3: Write in the decimal in the proper place according to place value.

Multiplication of Decimals:

Multiplication with decimals involves multiplying as usual. The tricky part is where to place the decimal. There is a rule on how to do it, but just in case, estimate the answer and that will help you with the placement of the decimal.

i.e. Multiply 7.28 x 5.3

Step 1: Estimate 7.28 x 5.3 will be between 7 x 5 =35 and 7 x 6 =42. Now you know that your answer has to be in this range or you have placed the decimal incorrectly.

Step 2: Multiply as you would with whole numbers, lining the numbers up on the right hand side.

$$\begin{array}{r} 7.28 \\ \times \quad 5.3 \\ \hline 2184 \\ +\ 36400 \\ \hline 38.584 \end{array}$$

Step 3: Place the decimal according to the rule: Count the total number of decimal places in the numbers being multiplied (7.28 and 5.3). There are 3 decimal places, so your answer should have 3 decimal places. Count back 3 places from the end of the number. We get 38.584.

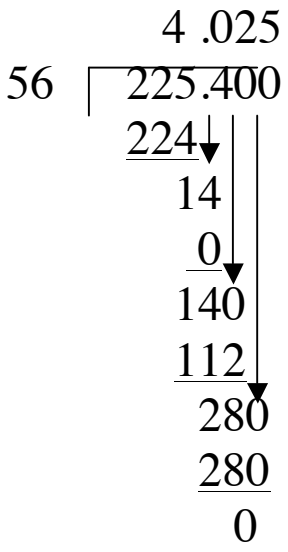
Step 4: Check that your answer makes sense with your estimate. The answer of 38.584 is in the range of 35 to 42, so your decimal is in the correct place.

Dividing Decimals:

Dividing Decimals by Whole Numbers i.e. 225.4 ÷ 56

Step 1: Set up your division so that the second number (divisor) is being divided into the first (dividend).

Step 2: Place the decimal point in the answer (quotient) directly above the one in the number being divided (dividend). Divide as usual.



Dividing Decimals by Decimals i.e. 201.24 ÷ 4.3

Step 1: Multiply the divisor and dividend by the same power of 10 to get rid of the decimal in the divisor. Here times 4.3 by 10 to get 43 and 201.24 by 10 to get 2012.4

Step 2: Place the decimal point in the answer (quotient) above the one in the divided.