

## Fraction Unit Notes

### Table of Contents:

<b>Topic</b>	<b>Page</b>
<b>Converting Decimals to Fractions and Fractions to Decimals</b>	<b>2</b>
<b>Fraction Vocabulary- Fraction, Part, Whole, Numerator, Denominator</b>	<b>3</b>
<b>Fraction Vocabulary-Unit Fraction, Equivalent Fractions and Common Denominators</b>	<b>4</b>
<b>Fraction Vocabulary-Simplify, Reduce</b>	<b>5</b>
<b>Fraction Vocabulary-Reciprocal</b>	<b>5</b>
<b>Types of Fractions (Proper, Improper, Mixed)</b>	<b>5</b>
<b>Changing from one Type of Fraction to another (Improper to Mixed, Mixed to Improper)</b>	<b>6</b>
<b>Operations on Fractions-Addition</b>	<b>8</b>
<b>Operations on Fractions-Subtraction</b>	<b>10</b>
<b>Operations on Fractions-Multiplication</b>	<b>11</b>
<b>Operations on Fractions-Division</b>	<b>13</b>
<b>Finding a Fractional Part of a Number</b>	<b>15</b>

## Converting Decimals to Fractions and Fractions to Decimals:

### Converting Decimals to Fractions

The easiest way to convert a decimal to a fraction is to read the decimal according to place value.

Example 1: Read 0.25 as “twenty five hundredths” . This gives us the fraction  $\frac{25}{100}$  . Then just reduce the fraction as needed.  $\frac{25}{100} = \frac{1}{4}$  .

Example 2: Read 0.3 as “three tenths” . This gives us the fraction  $\frac{3}{10}$  . It is already reduced to lowest terms.

### Converting Fractions to Decimals

To write a decimal for any fraction, divide the numerator by the denominator. This works because a fraction is a way of showing division.

$$\frac{3}{8} = 3 \div 8$$

$$\begin{array}{r} .375 \\ 8 \overline{) 3.000} \\ \underline{24} \phantom{00} \\ 60 \phantom{0} \\ \underline{56} \phantom{0} \\ 40 \phantom{0} \\ \underline{40} \\ 0 \end{array}$$

## **Vocabulary:**

### **Fraction**

A number that shows the relationship between a part and a whole.

### **Part**

A piece of the whole.

### **Whole**

An entire object.

In a fraction, the whole is divided into equal pieces.

### **Numerator**

The top number in a fraction.

The numerator counts the number of equal parts indicated by the fraction

For example, in the fraction  $\frac{3}{5}$ , the numerator shows that the fraction refers to 3 of the 5 equal parts that make up the whole.

### **Denominator**

The bottom number in a fraction.

The denominator represents the number of equal parts the whole has been divided into.

For example, in the fraction  $\frac{3}{5}$ , the denominator shows that the whole has been divided into 5 parts.

## **Unit Fraction**

A fraction with a numerator of 1. For example  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{8}$ ,  $\frac{1}{50}$  are unit fractions.

## **Equivalent**

They are equal in value. Equivalent fractions are fractions that have different numerators and denominators but which represent the same amount. For example,  $\frac{1}{2}$  and  $\frac{2}{4}$  are equivalent because they are both equal to 0.5. Equivalent fractions can be found by either multiplying or dividing the numerator and the denominator by the same number.

\*\*If you need to compare fractions in terms of their value, you need to rename them as equivalent fractions over the same denominator (common denominator).

Example 1: Which fraction is larger?  $\frac{4}{6}$  or  $\frac{15}{24}$

Step 1: Rename the fractions over the Lowest Common Multiple (LCM) of 6 and 24. (It is the lowest number that both 6 and 24 fit into). To find it first see if 6 fits into 24. Here it does so the LCM is 24. If this did not work, you would have to calculate the LCM first using our three step method from our *Number Sense and Decimal Unit Notes P.10* (staircase, venn diagram, multiply).

$$\frac{4}{6} = \frac{16}{24} \text{ because } 6 \times 4 = 24 \text{ and } 4 \times 4 = 16$$

so we are now comparing  $\frac{16}{24}$  and  $\frac{15}{24}$  so  $\frac{16}{24}$  or  $\frac{4}{6}$  is larger

## **Simplify/Reduce**

A simplified fraction is equivalent to the original fraction but has a smaller numerator and denominator. You reduce or simplify by dividing both the numerator and denominator by the same number.

Example 1: Reduce the following fraction:  $\frac{24}{36}$

Step 1: find a common factor to both: for example 2, 3, 4, 6, or 12. If you can think of the biggest factor (12 in this case), reducing will go faster, otherwise keep dividing both the numerator and denominator by the same number until you cannot divide anymore.

$\frac{24}{36} \div 2 = \frac{12}{18} \div 2 = \frac{6}{9} \div 3 = \frac{2}{3}$  stop at  $\frac{2}{3}$  because 2 and 3 have no common factors besides the number 1 so you cannot reduce it anymore.

To reduce faster, divide by the greatest common factor from the start.  $\frac{24}{36} \div 12 = \frac{2}{3}$ . Notice you get the same answer as above, but with fewer calculations.

## **Reciprocal**

Two numbers that have a product of one.

For example  $\frac{3}{4}$  and  $\frac{4}{3}$  are reciprocals because  $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

## **Types of Fractions:**

### **Proper Fractions**

A proper fraction is a fraction that has a top number (numerator) that is smaller than the bottom number (denominator). Proper fractions represent quantities that

are less than 1 whole. The decimal equivalents are between 0 and 1.

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$$

### **Improper Fractions**

A improper fraction is a fraction that has a top number (numerator) that is larger than the bottom number (denominator). Improper fractions represent quantities that are greater than 1 whole. The decimal equivalents are larger than 1.0.

$$\frac{8}{3}, \frac{4}{2}, \frac{9}{5}$$

### **Mixed Fractions (Mixed Number)**

A mixed number is a whole number and a proper fraction combined. Mixed fractions represent quantities that are greater than 1 whole. The decimal equivalents are larger than 1.0.

$$3\frac{1}{2}, 4\frac{1}{4}, 5\frac{1}{3}$$

## **Changing From One Type of Fraction to Another:**

### **Mixed Fraction to Improper Fraction**

Example 1: Write  $2\frac{1}{4}$  as an improper fraction.

Steps:

1-Write the whole number part (the 2 wholes) as a fraction  $\frac{8}{4}$

2-Add this to the proper fraction part ( $\frac{1}{4}$ ) So  $\frac{8}{4} + \frac{1}{4} = \frac{9}{4}$

OR:

1-Multiply the whole number part by the fraction's denominator. ( $2 \times 4 = 8$ )

2-Add that to the numerator ( $8+1=9$ )

3-Write that result on top of the denominator. ( $\frac{9}{4}$ )

### **Improper Fraction to Mixed Fraction**

Example 1: Write  $\frac{9}{4}$  as an improper fraction.

Steps:

1-Divide the top number (numerator) by the bottom number (denominator). Stop when you have a whole number and a remainder.

2-The whole number becomes the whole number part of the Mixed Fraction and the remainder will be the proper fraction part of the Mixed Fraction.

$$\begin{array}{r} 2 \quad \text{R1} \\ 4 \overline{) 9.000} \\ \underline{8} \\ 1 \end{array}$$

The remainder becomes the numerator in the proper fraction. The denominator is the same as in the improper fraction

So  $\frac{9}{4} = 2\frac{1}{4}$

## **Operations on Fractions:**

It is important to learn how to add, subtract, multiply, and divide fractional numbers.

## **Common Denominators:**

For adding and subtracting fractions, you need to find a common denominator for the fractions before you can add or subtract. This means that you need to rename the fractions as equivalent fractions with the same denominator. (See Page 4: Equivalent fractions)

## **Adding Fractions**

### **Example 1:**

### **Adding Proper Fractions**

$$= \frac{2}{4} + \frac{3}{8}$$

$$= \frac{4}{8} + \frac{3}{8}$$

$$= \frac{7}{8}$$

1) Rename the fractions with common denominators

2) Add the numerators and place

over the denominator. Do not add the denominators.

3) Reduce the fraction if possible.



### **Example 2:**

#### **Adding Improper Fractions**

$$= \frac{9}{4} + \frac{8}{3}$$

$$= \frac{27}{12} + \frac{32}{12}$$

$$= \frac{59}{12}$$

$$= 4\frac{11}{12}$$

- 1) Rename the fractions with common denominators
- 2) Add the numerators and place over the denominator. Do not add the denominators.
- 3) Rename the answer as a mixed number.
- 4) Reduce the fraction if possible.

### **Example 3:**

#### **Adding Mixed Numbers**

$$= 3\frac{2}{3} + 4\frac{4}{6}$$

$$= \frac{11}{3} + \frac{28}{6}$$

$$= \frac{22}{6} + \frac{28}{6}$$

$$= \frac{50}{6}$$

$$= 8\frac{2}{6}$$

$$= 8\frac{1}{3}$$

- 1) Rename the fractions as improper fractions.
- 2) Rename the fractions with common denominators
- 3) Add the numerators and place over the denominator. Do not add the denominators.
- 4) Rename the answer as a mixed number.
- 5) Reduce the fraction if possible.

#### **Subtracting Fractions**

**Example 1:**  
**Subtracting Proper Fractions**

$$\begin{aligned} &= \frac{8}{12} - \frac{2}{6} && 1) \text{ Rename the fractions with common denominators} \\ &= \frac{8}{12} - \frac{4}{12} && 2) \text{ Subtract the numerators and place} \\ &= \frac{4}{12} && \text{over the denominator. Do not subtract the} \\ &= \frac{1}{3} && \text{denominators.} \\ & && 3) \text{ Reduce the fraction if possible.} \end{aligned}$$

**Example 2:**  
**Subtracting Improper Fractions**

$$\begin{aligned} &= \frac{8}{3} - \frac{9}{4} && 1) \text{ Rename the fractions with common denominators} \\ &= \frac{32}{12} - \frac{27}{12} && 2) \text{ Subtract the numerators and place} \\ &= \frac{5}{12} && \text{over the denominator. Do not subtract the} \\ & && \text{denominators.} \\ & && 3) \text{ Reduce the fraction if possible.} \end{aligned}$$

**Example 3:**  
**Subtracting Mixed Numbers**

$$\begin{aligned} &= 4\frac{2}{8} - 9\frac{1}{4} && 1) \text{ Rename the fractions as improper fractions.} \\ &= \frac{34}{8} - \frac{9}{4} && 2) \text{ Rename the fractions with common denominators} \\ &= \frac{34}{8} - \frac{18}{8} && 3) \text{ Subtract the numerators and place} \\ &= \frac{16}{8} && \text{over the denominator. Do not subtract the} \\ &= \frac{2}{1} && \text{denominators.} \\ &= 2 && 4) \text{ Rename the answer as a mixed number.} \\ & && 5) \text{ Reduce the fraction if possible.} \end{aligned}$$

## Multiplying Fractions

### Example 1:

#### Multiplying Proper Fractions

$$= \frac{3}{4} \times \frac{4}{5}$$

$$= \frac{12}{20}$$

$$= \frac{3}{5}$$

- 1) Multiply the numerator of each fraction together and this will become the numerator in your answer.
- 2) Multiply the denominator of each fraction together and this will become the denominator in your answer.
- 3) Reduce the fraction if possible.

### Example 2:

#### Multiplying Improper Fractions

$$= \frac{9}{4} \times \frac{8}{3}$$

$$= \frac{72}{12}$$

$$= 6$$

- 1) Multiply the numerator of each fraction together and this will become the numerator in your answer.
- 2) Multiply the denominator of each fraction together and this will become the denominator in your answer.
- 3) Reduce the fraction if possible. ( $72 \div 12 = 6$ )

### Example 3:

#### Multiplying Mixed Numbers

$$= 4\frac{2}{3} \times 3\frac{1}{4}$$

$$= \frac{14}{3} \times \frac{13}{4}$$

$$= \frac{182}{12}$$

$$= 15\frac{2}{12}$$

- 1) Rename the fractions as improper fractions
- 2) Multiply the numerator of each fraction together and this will become the numerator in your answer.
- 3) Multiply the denominator of each fraction together and this will become the denominator in your answer.
- 4) Reduce the fraction if possible.

$$= 15\frac{1}{6}$$

### **Cross Simplifying to Reduce before you Multiply**

If you have fractions that can be simplified you can do so before you multiply to avoid having such big numbers to reduce afterwards.

Example 1: Multiply  $\frac{12}{18} \times \frac{12}{24} =$

$$= \frac{12}{18} \times \frac{12}{24} = \frac{144}{432} \text{ so to simplify...}$$

$$= \frac{144}{432} \div 2 = \frac{72}{216} \div 2 = \frac{36}{108} \div 2 = \frac{18}{54} \div 2 = \frac{9}{27} \div 9 = \frac{1}{3}$$

If you follow the multiplication instructions above (numerator x numerator and denominator x denominator) you will get a fraction answer  $\frac{144}{432}$  that has big numbers that you will need to reduce. This will take several steps to arrive at the answer in lowest terms.

\*\*Instead of doing this long process, it is easier to cross simplify before you multiply. You will also have a better chance of getting the answer correct because there are less steps.

Example 1: Multiply  $\frac{12}{18} \times \frac{12}{24} =$  You want to simplify

diagonally before multiplying. If you look at the 12 and the 24, you can see that 12 is a factor of both the numbers so you can divide each by 12. Now you would have  $\frac{1}{18} \times \frac{12}{2} =$  .

If you now look at the 18 and the 12, you can see that 6 is a factor of both the numbers so you can divide them each

by 6. Now you will have  $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ . Now just multiply as usual.  $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ . This is much easier to reduce:  $\frac{2}{6} \div 2 = \frac{1}{3}$

## Dividing Fractions

In order to divide fractions, you need to multiply by the reciprocal.

Reciprocals are two numbers that have a product of one.

For example  $\frac{3}{4}$  and  $\frac{4}{3}$  are reciprocals because  $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

To find the reciprocal of any number, flip the fraction over. If you have a whole number or a mixed number, rewrite it as a fraction, then flip it.

i.e.  $2 = \frac{2}{1}$  so then the reciprocal of 2 is  $\frac{1}{2}$

i.e.  $3 \frac{2}{3} = \frac{11}{3}$  so then the reciprocal of  $\frac{11}{3}$  is  $\frac{3}{11}$

### Example 1:

#### Dividing a Whole Number by a Fractions

$4 \div \frac{2}{3}$ $= \frac{4}{1} \div \frac{2}{3}$ $= \frac{4}{1} \times \frac{3}{2}$ $= \frac{12}{2}$ $= 6$	<ol style="list-style-type: none"> <li>1) Write the whole number as a fraction</li> <li>2) Use the KCF method               <ul style="list-style-type: none"> <li><b>K</b>-eep (the first fraction).</li> <li><b>C</b>-hange (change the division to multiplication).</li> <li><b>F</b>-lip (the second fraction –use reciprocal).</li> </ul> </li> <li>3) Multiply by the reciprocal of 2<sup>nd</sup> fraction</li> <li>4) Reduce the fraction if possible.</li> </ol>
---	---

### **Example 2:**

#### **Dividing a Fraction by a Whole Number**

$$\begin{aligned} & \frac{7}{8} \div 3 \\ = & \frac{7}{8} \div \frac{3}{1} \\ = & \frac{7}{8} \times \frac{1}{3} \\ = & \frac{7}{24} \end{aligned}$$

- 1) Write the whole number as a fraction
- 2) Use the KCF method  
**K**-eep (the first fraction).  
**C**-hange (change the division to multiplication).  
**F**-lip (the second fraction –use reciprocal).
- 3) Multiply by the reciprocal of 2<sup>nd</sup> fraction
- 4) Reduce the fraction if possible.

### **Example 3:**

#### **Dividing a Fraction by a Fraction**

$$\begin{aligned} & \frac{3}{4} \div \frac{1}{8} \\ = & \frac{3}{4} \times \frac{8}{1} \\ = & \frac{24}{4} \\ = & 6 \end{aligned}$$

- 1) Use the KCF method  
**K**-eep (the first fraction).  
**C**-hange (change the division to multiplication).  
**F**-lip (the second fraction –use reciprocal).
- 2) Multiply by the reciprocal of 2<sup>nd</sup> fraction
- 3) Reduce the fraction if possible.

### **Example 4:**

### **Dividing Mixed Numbers by Mixed Numbers**

$$\begin{aligned} & 4\frac{3}{4} + 1\frac{1}{2} \\ &= \frac{19}{4} \div \frac{3}{2} \\ &= \frac{19}{4} \times \frac{2}{3} \\ &= \frac{38}{12} \\ &= 3\frac{2}{12} \\ &= 3\frac{1}{6} \end{aligned}$$

- 1) Change the mixed numbers into improper fractions
- 2) Use the KCF method  
**K**-eep (the first fraction).  
**C**-hange (change the division to multiplication).  
**F**-lip (the second fraction –use reciprocal).
- 3) Multiply by the reciprocal of 2<sup>nd</sup> fraction
- 4) Reduce the fraction if possible.

### **Finding a Fractional Part of a Number**

Find  $\frac{2}{3}$  of 36

The word "of" means multiply

$$\begin{aligned} &= \frac{2}{3} \times 36 \\ &= \frac{2}{3} \times \frac{36}{1} \\ &= \frac{72}{3} \\ &= 24 \end{aligned}$$

- 1) The word of means to multiply
- 2) Change the whole number to a fraction
- 3) Multiply the Fractions.
- 4) Divide the numerator by the denominator the get the fractional part of the number.

**OR**

$\frac{2}{3}$  of 36

- 1) Find  $\frac{1}{3}$  of 36 by doing  $36 \div 3$ . So  $\frac{1}{3}$  of 36 = 12
- 2) Double the answer for  $\frac{1}{3}$  to find  $\frac{2}{3}$ . So  $12 \times 2 = 24$