Algebraic Equations

1. **Definitions:**
   Equality: A true or false statement that uses the " = " symbol.

   Equation: An equality containing at least one variable.

   Domain or replacement set of variables: The set of values that can replace a variable in an equation. **When you solve an equation the answer is the domain.

   Solving an equation: Finding the values of the domain with which the variable(s) must be replaced to obtain a true equality.

   Solution of an equation: The domain value that transforms an equation into a true equality.

   Solution Set: Set of Solutions.

2. **Solving Equations:**
   Equations are equalities involving variables. When transforming equalities, the main purpose is to determine the value of the variable. When transforming equations keep in mind four general rules. **The solutions of an equation are unchanged if:**
   1- the same number is added to both sides of the equation.
   2- the same number is subtracted from both sides of the equation.
   3- both sides of the equation are multiplied by the same number (other than zero)
   4- both sides of the equation are divided by the same number (other than zero).
3. **Types of Equations:**
   The following examples are 4 different types of equations that you will encounter along with step-by-step methods for solving them. Although the basic idea of isolating the variable is the same throughout, the order of the steps are a bit different. Use your notes as a guide until you feel confident solving the various types of equations on your own.

Show all your work as shown below.

TYPE 1: ONE-STEP EQUATION

	$x + 5 = 17$
S5	$x + 5 - \mathbf{5} = 17 - \mathbf{5}$
CT	$x = 12$

- Isolate variable by subtracting 5 from both sides
- Collect terms.

ONE-STEP CROSS MULTIPLICATION

	$\frac{P}{4} = 20$
CM	$p = 20\ (4)$
CT	$p = 80$

- Cross Multiply to eliminate fraction.
- Collect terms.

TYPE 2: TWO-STEP EQUATION

	$6y + 4 = 40$
S4	$6y + 4 - \mathbf{4} = 40 - \mathbf{4}$
CT	$\frac{6y}{\mathbf{6}} = \frac{36}{\mathbf{6}}$
D6	
ANS	$y = 6$

- Isolate variable by subtracting 4 from both sides
- Collect terms.
- Divide both sides by 6.

TWO-STEP CROSS-MULTIPLICATION

	$\frac{3x}{4} + 5 = 11$
S5	$\frac{3x}{4} + 5 - \mathbf{5} = 11 - \mathbf{5}$
CT	$\frac{3x}{4} = 6$
CM	$3x = 6\ (4)$
CT	$\frac{3x}{\mathbf{3}} = \frac{24}{\mathbf{3}}$
D3	
ANS	$x = 8$

- Isolate fraction by subtracting 5 from both sides of equation.
- Collect terms.
- Cross multiply to eliminate fraction.
- Divide by numerical coefficient of variable (3)

TYPE 3: VARIABLES ON BOTH SIDES

	$6x + 3 = x + 8$
S3	$6x + 3 - \mathbf{3} = x + 8 - \mathbf{3}$
CT	$6x = x + 5$
Sx	$6x - \mathbf{x} = x + 5 - \mathbf{x}$
CT	$\frac{5x}{\mathbf{5}} = \frac{5}{\mathbf{5}}$
D5	
ANS	$x = 1$

- Isolate variable by subtracting 3 from both sides
- Collect terms.
- Subtract x from both sides so all variables are on one side of the equation.
- Collect terms.
- Divide by numerical coefficient of variable (5)

NEEDS SIMPLIFICATION

	$10x + 6x - 4x + 2 = 146$
CT	$12x + 2 = 146$
S2	$12x + 2 - \mathbf{2} = 146 - \mathbf{2}$
CT	$\frac{12x}{\mathbf{12}} = \frac{144}{\mathbf{12}}$
D12	
ANS	$x = 12$

- Simplify by Collecting like terms.
- Isolate variable by subtracting 2 from both sides of equation.
- Collect terms.
- Divide by numerical coefficient of variable (12)

DISTRIBUTION

	$12(x + 4) = 10(x - 4)$
CT	$12x + 48 = 10x - 40$
S2	$12x + 48 - \mathbf{48} = 10x - 40 - \mathbf{48}$
CT	$12x = 10x - 88$
S10x	$12x - \mathbf{10x} = 10x - 88 - \mathbf{10x}$
CT	$\frac{2x}{\mathbf{2}} = \frac{-88}{\mathbf{2}}$
D12	
ANS	$x = -44$

- Distribute to remove brackets (multiply each part inside brackets by # outside brackets)
- Isolate variable by subtracting 48 from both sides of equation.
- Collect terms.
- Bring all variables to one side by subtracting 10x from both sides.
- Collect like terms
- Divide by numerical coefficient of variable (2)

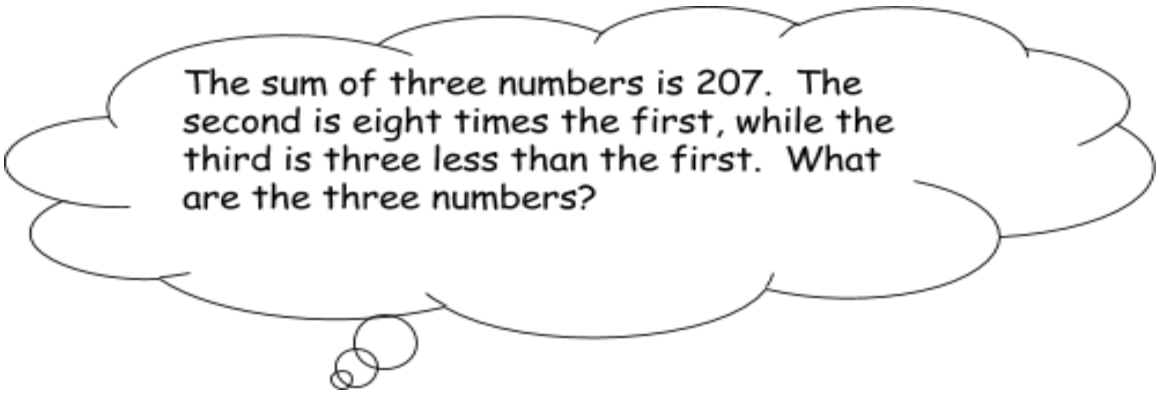
DOUBLE FRACTION

	$\frac{4x}{6} + \frac{2x}{8} = 44$
M24	$\mathbf{24}\frac{4x}{6} + \mathbf{24}\frac{2x}{8} = \mathbf{24}(44)$
RB	$\frac{96x}{6} + \frac{48x}{8} = 1056$
SIM	$16x + 6x = 1056$
CT	$\frac{22x}{\mathbf{22}} = \frac{1056}{\mathbf{22}}$
D22	
ANS	$x = 48$

- Multiply each part of the equation by the (LCM) lowest common multiple of the denominators. Here the LCM of 6 and 8 is 24.
- Simplify each fraction by dividing by denominator.
- Collect Terms
- Divide by numerical coefficient of variable (22)

4. **Problem Solving Strategy:**

Use the following method to solve problems where you need to identify variables, set up an equation, and solve the equation. There are six steps.



STEP #1 Set up an equation using words or blanks.

i.e. number 1 + number 2 + number 3 = 207

STEP #2 Decide which number is represented by (x). It is the one that you know the least about. Decide algebraic expressions for the other numbers.

i.e. 1^{st} number = x
2^{nd} number = 8x
3^{rd} number = x - 3

STEP #3 Set up equation by filling in blanks from step #1.

i.e. $\underline{x} + \underline{8x} + \underline{x-3} = 207$

STEP #4 Collect terms and solve the equation

i.e. $x + 8x + x-3 = 207$

CT	$10x - 3 = 207$
A3	$10x - 3 + \mathbf{3} = 207 + \mathbf{3}$
CT	$\underline{10x} = \underline{210}$
D10	$\mathbf{10} \quad \mathbf{10}$
ANS	$x = 21$

STEP #5 Use the value of (x) that you find to determine the other two numbers

i.e. x = 21 so 8x =8(21) =168 and x-3 =21 - 3 =18

STEP #6 Check your answer. Does 168 + 21 + 18 = 207. If yes, the numbers are 21, 168, and 18 (assuming you did steps 1 and 4 correctly).